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Dynamic Modeling and Controller Design for a Seamless Two-Speed Transmission for Electric Vehicles*

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***Abstract*— Transmission is one of the crucial elements of the driveline that affects vehicle fuel economy and comfort. It can transfer power in different combinations of torque and speed. This paper focuses on the modeling, simulation and control of a two-speed transmission for electric vehicles which has seamless gear shifting specification. The transmission incorporates two-stage planetary gear sets and two braking mechanisms to control the gear shifting. The dynamic model is developed by using the kinematic equations of the planetary gear trains and the Euler-Lagrange equations to derive the equations of motion. The mathematical model is validated by using the SimDriveLine library of MATLAB/Simulink®. The controller design employs optimal control methods to provide seamless shifting with minimum transition time. Then, by relaxing ideal constraints, a feasible controller is designed based on input-output and input-state feedback linearization. Simulation results demonstrate the ability of the proposed transmission to have smooth shifting without excessive oscillations in the output torque and speed.**

I. INTRODUCTION

Increasing fuel cost and environmental concerns have pushed the automotive industry to gradually replace internal combustion engine (ICE) vehicles with hybrid electric (HEV) and fully electric vehicles (EV). However, the energy density of electric batteries is much less than that of fossil fuels. So, by changing the source of power from internal combustion engine to electrical motor, it is required to minimize the losses in the driveline in order to maximize the range of EV's. The transmission is one of the most important parts of the driveline.

The most common types of transmissions of vehicles are Manual Transmission (MT), Automated Manual Transmission (AMT), Automatic Transmission (AT), Dual Clutch Transmission (DCT) and Continuously Variable Transmission (CVT) [1]. In general, the advantages of these transmissions can be classified as in Table I.

Most transmissions currently used for EV's were initially designed for ICE vehicles. Since ICE cannot operate below certain speeds and their speed control during gear changes is not an easy task, the presence of clutches or torque convertors are inevitable for start ups, idle running and gear changing. This, however, is not the case for EV's as electric motors are speed controllable in a wide range of operating speeds. This difference provides an opportunity for designing novel transmissions. In this paper a transmission is proposed as an attempt to achieve all the required features of an ideal transmission for EVs. The proposed transmission is comprised of a dual-stage planetary gear set with common ring and common sun gears. The ratio of the pitch diameter of the ring gear to the sun gear in the input and output sides are different in order to have two different gear ratios. Two friction brakes are used to control the flow of power during gearshift to have a fast and smooth gear change. These two friction brakes control the speed of the sun and the ring gears. Fig.1 shows the schematic view of the proposed transmission. According to Fig.1, the input of the transmission is the carrier on the left hand side of the figure, which is directly attached to an electric motor. The output of the mechanism is the carrier on the right hand side which is attached to the load. Two different gear ratios can be obtained by fixing the sun or the ring gears. Furthermore, by controlling the brakes, the gear shifting can be made seamless and without any torque interruption. Here, for brevity, sun, ring and planets are used instead of sun gear, ring gear and planet gears, respectively.

Since the set of efficient operating points for electric motors is rich enough, multiplicity of gear numbers and continuously variable transmissions (CVT)[2],[3] are not necessary. The proposed transmission employs planetary gear sets which have a high power density as the torque is distributed over several gears [4]. This design shares the benefits of DCT's [5] and in addition, it is more compact which makes it a better alternative to become a commercial transmission. A feature of special interest in DCT is the elimination of output torque interruptions during gear changing, in contrast to AMT's [6],[7],[8] where the driveline is disengaged from and re-engaged to the traction motor or engine which reduces passenger comfort and lifetime of the synchronizer. Shift control in DCTs consists of two different phases (the same as ATs [9]) called the torque phase and the inertia phase. While a similar method can be implemented on the proposed transmission, in this paper a control method specific to the system's model is developed.

TABLE I
ADVANTAGES OF DIFFERENT TRANSMISSIONS

Transmissions	Ease of Use	Reliability	No Torque Interruption	Fuel Efficiency	Commercial Vehicles
MT		●		●	●
AMT	●			●	●
AT	●	●	●		
DCT	●	●	●	●	
CVT	●	●	●		

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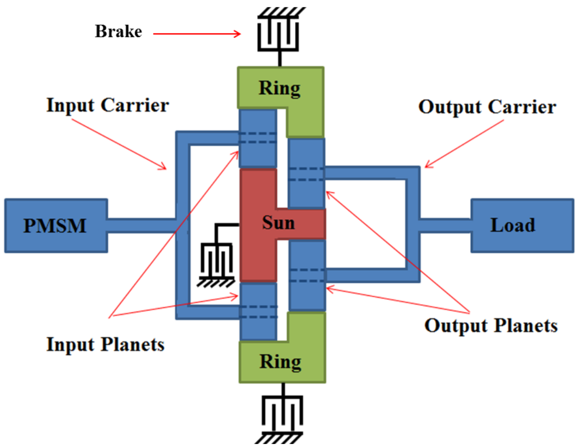

Fig. 1. Schematic view of the Two Speed Automated Transmission

II. KINEMATIC ANALYSIS OF THE PROPOSED TRANSMISSION

A. Kinematic Equations

As the planetary gear trains are the main components of the proposed transmission, the required kinematic equations to derive the equations of motion are reviewed. The kinematic relations between planetary gear components such as Carrier (C), Sun(S), Planets (P) and Ring (R) are [10]:

$$r_R\omega_R = r_P\omega_P + r_C\omega_C; \quad r_R = r_P + r_C \tag{1}$$

$$r_C\omega_C = r_P\omega_P + r_S\omega_S; \quad r_C = r_P + r_S \tag{2}$$

Where r_S, r_P and r_R are the pitch radius of the sun, planet and ring, respectively. The parameter r_C is the radius of the circle on which the planets are mounted. The parameters ω_S, ω_P, ω_R and ω_C are the angular velocity of the sun, planets, ring and carrier, respectively. By eliminating ω_P and r_P from (1) and (2), the kinematic relation between the Ring, Sun and Carrier is as follows:

$$(r_R + r_S)\omega_C = r_S\omega_S + r_R\omega_R \tag{3}$$

For simplification of the formulation, the ratio of the pitch radius of the ring r_R to the sun r_S in the input and the output of the transmission are considered as:

$$R_1 := \left(\frac{r_R}{r_S}\right)_{input} ; \quad R_2 := \left(\frac{r_R}{r_S}\right)_{output} \tag{4}$$

It is obvious that R_1 and R_2 are greater than one because the pitch radius of the ring is always greater than the sun's.

During gear changing, the system has two degrees of freedom. Hence, it is required to select two generalized coordinates to derive the equations of motion. In this paper, the generalized coordinates are chosen to be ω_S and ω_R and all the angular velocities need to be expressed as functions of ω_S and ω_R. From (1)-(4) it can be concluded that:

$$\begin{cases} \omega_{C,in} = \dfrac{R_1\omega_R + \omega_S}{(R_1+1)}; & \omega_{C,out} = \dfrac{R_2\omega_R + \omega_S}{(R_2+1)} \\ \omega_{P,in} = \dfrac{R_1\omega_R - \omega_S}{(R_1-1)}; & \omega_{P,out} = \dfrac{R_2\omega_R - \omega_S}{(R_2-1)} \end{cases} \tag{5}$$

B. Gear Ratios

According to equations set (5), the speed ratio of the input of the transmission to the output can be expressed as follows:

$$\frac{\omega_{C,out}}{\omega_{C,in}} = \frac{(R_1+1)(\omega_S + R_2\omega_R)}{(R_2+1)(\omega_S + R_1\omega_R)} \tag{6}$$

According to (6), three different gear ratios are achievable:

1) If the ring is completely grounded ($\omega_R = 0$):

$$\frac{\omega_{C,out}}{\omega_{C,in}} = \frac{(R_1+1)}{(R_2+1)} = GR_1 \tag{7}$$

2) If the sun is completely grounded ($\omega_S = 0$):

$$\frac{\omega_{C,out}}{\omega_{C,in}} = \frac{(R_1+1)R_2}{(R_2+1)R_1} = GR_2 \tag{8}$$

3) If neither sun nor ring are completely grounded ($\omega_R \neq 0$ and $\omega_S \neq 0$):

$$\frac{\omega_{C,out}}{\omega_{C,in}} = \frac{(R_1+1)(\omega_S + R_2\omega_R)}{(R_2+1)(\omega_S + R_1\omega_R)} = GR_T \tag{9}$$

Here, GR_1 and GR_2 are considered as the first and the second gear ratios where GR_T is the transient gear ratio from the first gear ratio to the second one during gear shifting. Even though the gear ratios are dependent, it is possible to solve (7) and (8) for R_1 and R_2 in order to get the desired GR_1 and GR_2.

III. DYNAMIC MODELING AND VALIDATION

The vector of independent generalized coordinates q, is considered as $q = [\omega_S \; \omega_R]^T$. The Euler-Lagrange equations for a minimum set of generalized coordinates are as follows:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = Q_k^{appl,nc} \tag{10}$$

Where $L = T - V$ is the Lagrangian, T is the total kinetic energy and V is the total potential energy of the system.

By considering the centre of mass of the system as the reference point for the gravitational energy and by considering all the mechanical parts rigid, the total potential energy of the system becomes zero ($V = 0$). The kinetic energy of the system consists of the kinetic energy of the input and output carriers, ring, sun, input and output planets as follows:

$$\begin{aligned} T = {} & \overbrace{\frac{1}{2}I_{C,in}\omega_{C,in}^2}^{Input\ Carrier} + \overbrace{\frac{1}{2}I_{C,out}\omega_{C,out}^2}^{Output\ Carrier} + \overbrace{\frac{1}{2}I_R\omega_R^2}^{Ring\ Gear} \\ & + \overbrace{\frac{1}{2}I_S\omega_S^2}^{Sun\ Gear} + \overbrace{4\left(\frac{1}{2}I_{P,in}\omega_{P,in}^2 + \frac{1}{2}m_{P,in}r_{C,in}^2\omega_{C,in}^2\right)}^{Four\ Input\ Planet\ Gears} \\ & + \overbrace{4\left(\frac{1}{2}I_{P,out}\omega_{P,out}^2 + \frac{1}{2}m_{P,out}r_{C,out}^2\omega_{C,out}^2\right)}^{Four\ Output\ Planet\ Gears} \end{aligned} \tag{11}$$

where $I_{C,in}$, $I_{C,out}$, I_S, I_R, $I_{P,in}$ and $I_{P,out}$ are the moment of inertia of the input carrier, output carrier, sun, ring, input planets and output planets, respectively. $m_{P,in}$ and $m_{P,out}$ are the mass of the input and output planets. Now, all the other velocities should be written as functions of these two general coordinates (ω_R and ω_S) as the following:

$$
\begin{aligned}
T = &\frac{1}{2}(I_{C,in} + 4m_{P,in}r_{C,in}^2)\,(\overbrace{\frac{\omega_S^2 + R_1^2\omega_R^2 + 2R_1\omega_R\omega_S}{(R_1+1)^2}}^{\omega_{C,in}})\\
&+ \frac{1}{2}(I_{C,out} + 4m_{P,out}r_{C,out}^2)\,(\overbrace{\frac{\omega_S^2 + R_2^2\omega_R^2 + 2R_2\omega_R\omega_S}{(R_2+1)^2}}^{\omega_{C,out}})\\
&+ 4\{\frac{1}{2}I_{P,in}\,(\overbrace{\frac{\omega_S^2 + R_1^2\omega_R^2 - 2R_1\omega_R\omega_S}{(R_1-1)^2}}^{\omega_{P,in}})\} + \frac{1}{2}I_S\omega_S^2\\
&+ 4\{\frac{1}{2}I_{P,out}\,(\overbrace{\frac{\omega_S^2 + R_2^2\omega_R^2 - 2R_2\omega_R\omega_S}{(R_2-1)^2}}^{\omega_{P,out}})\} + \frac{1}{2}I_R\omega_R^2
\end{aligned}
\tag{12}
$$

By using the Euler-Lagrange equations, the equations of motion for the two generalized coordinates $q = [\omega_S\ \omega_R]^T$ can be written as follows:

$$
\begin{cases}
\dot{\omega}_S = \frac{1}{a}(T_{BS}\tau - T_{BR}\lambda - \omega_S C_S\tau + \omega_R C_R\lambda + cT_M\\
\quad + dT_{load} + T_{Sf}\tau - T_{Rf}\lambda)\\
\dot{\omega}_R = \frac{1}{a}(T_{BR}\gamma - T_{BS}\lambda + \omega_S C_S\lambda - \omega_R C_R\gamma + eT_M\\
\quad + fT_{load} + T_{Rf}\gamma - T_{Sf}\lambda
\end{cases}
\tag{13}
$$

Where the coefficients are:

$$
\begin{cases}
\gamma = [\alpha + \beta + I_S + \frac{4I_{P,in}}{(R_1-1)^2} + \frac{4I_{P,out}}{(R_2-1)^2}]\\
\lambda = [\alpha R_1 + \beta R_2 - \frac{4I_{P,in}R_1}{(R_1-1)^2} - \frac{4I_{P,out}R_2}{(R_2-1)^2}]\\
\tau = [\alpha R_1^2 + \beta R_2^2 + I_R + \frac{4I_{P,in}R_1^2}{(R_1-1)^2} + \frac{4I_{P,out}R_2^2}{(R_2-1)^2}]\\
a = (\gamma\tau - \lambda^2);\ c = \frac{\tau - R_1\lambda}{R_1+1};\ d = \frac{\tau - R_2\lambda}{R_2+1}\\
e = \frac{\gamma R_1 - \lambda}{R_1+1};\ f = \frac{\gamma R_2 - \lambda}{R_2+1}
\end{cases}
$$

$$
\begin{cases}
\alpha = (I_{C,in} + 4m_{P,in}r_{C,in}^2)/(R_1+1)^2\\
\beta = (I_{C,out} + 4m_{P,out}r_{C,out}^2)/(R_2+1)^2
\end{cases}
$$

where C_S, C_R, T_{Sf} and T_{Rf} are related to the Coulomb and viscous friction of the transmission and can be measured from experimental tests. Dynamics of the load can be determined from longitudinal dynamics of vehicle and be expressed as the following function [11]:

$$
T_{load} = I(\omega_{C,out}, \phi, C_d, C_r, A) \tag{14}
$$

where ϕ is the road angle, C_d is the aerodynamic drag coefficient, C_r is the rolling resistance coefficient and A is the frontal area. A model of the proposed transmission has been built in MATLAB/Simulink® by using SimDriveLine to validate the performance of controllers which are design in the next sections.

IV. Controller Design

In this section, ideal and feasible controllers are presented. The control variables are T_M, T_{BS} and T_{BR} and the objectives are constant output speed and output torque during the gear change as well as smooth gear ratio control and avoidance of power interruption during gear shifting process. The optimal controller is designed based on the representation of the desired objectives as mathematical constraints. The feasible controller is then derived by relaxation of the mathematical constraints from equalities to controlled values.

A. Ideal Controller

As explained earlier, the main control objectives is to provide constant output speed and torque while minimizing the shifting time during gear changing process. Here, (13) is represented as:

$$
\begin{aligned}
\begin{bmatrix} \dot{\omega}_S \\ \dot{\omega}_R \end{bmatrix} = &\tfrac{1}{a}\begin{bmatrix} -C_S\tau & C_R\lambda \\ C_S\lambda & -C_R\gamma \end{bmatrix}\begin{bmatrix} \omega_S \\ \omega_R \end{bmatrix}\\
&+\tfrac{1}{a}\begin{bmatrix} \tau & -\lambda & c \\ -\lambda & \gamma & e \end{bmatrix}\begin{bmatrix} T_{BS} \\ T_{BR} \\ T_M \end{bmatrix}\\
&+\tfrac{1}{a}\begin{bmatrix} \tau & -\lambda & d \\ -\lambda & \gamma & f \end{bmatrix}\begin{bmatrix} T_{Sf} \\ T_{Rf} \\ T_{load} \end{bmatrix}
\end{aligned}
\tag{15}
$$

Given that T_{Sf} and T_{Rf} have known fixed values and T_{load} is known from (14), the above system can easily be shown to be controllable. In addition, since the output speed is geometrically related to the car's speed, which is continuously measured, the system is also observable as the observability matrix for the following output equation has a full rank.

$$
\omega_{C,out} = \begin{bmatrix} \frac{1}{R_2+1} & \frac{R_2}{R_2+1} \end{bmatrix}\begin{bmatrix} \omega_S \\ \omega_R \end{bmatrix} \tag{16}
$$

The objective of the control is to go from one gear to another (i.e. from (7) to (8) through (9) and vice versa) by means of engaging and releasing the brakes. Thus initial and terminal conditions (15) can be expressed as

$$
\begin{bmatrix} \omega_S \\ \omega_R \end{bmatrix} = \begin{bmatrix} \omega_S\,(GR_1) \\ 0 \end{bmatrix} \underset{downshift}{\overset{upshift}{\rightleftharpoons}} \begin{bmatrix} \omega_S \\ \omega_R \end{bmatrix} = \begin{bmatrix} 0 \\ \omega_R\,(GR_2) \end{bmatrix} \tag{17}
$$

As mentioned in the control objectives, it is desired to maintain the output speed and output torque during the gear change. This requirement is ideally interpreted as

$$
\dot{\omega}_{C,out} = 0 \Rightarrow \dot{\omega}_R = \frac{-1}{R_2}\dot{\omega}_S \tag{18}
$$

Substituting (18) in (15) gives:

$$
\begin{aligned}
(\lambda R_2 - \tau)\,(C_S\omega_S - T_{BS}) - (\gamma R_2 - \lambda)\,(C_R\omega_R - T_{BR})\\
+ (c + eR_2)\,T_M - (\lambda R_2 - \tau)\,T_{Sf} + (\gamma R_2 - \lambda)\,T_{Rf}\\
+ (d + fR_2)\,T_{load} = 0
\end{aligned}
\tag{19}
$$

This constraint reduces the number of independent control inputs to two, giving the third input as a dependent variable to maintain the speed. Taking

$$T_M = \tfrac{1}{c+eR_2}\left(\left[\begin{array}{cc}\lambda R_2-\tau & \lambda-\gamma R_2\end{array}\right]\left[\begin{array}{c}T_{BS}\\ T_{BR}\end{array}\right]\right.$$
$$+\left[\begin{array}{cc}(\tau-\lambda R_2)\,C_S & (\gamma R_2-\lambda)\,C_R\end{array}\right]\left[\begin{array}{c}\omega_S\\ \omega_R\end{array}\right]$$
$$\left.+(\lambda R_2-\tau)\,T_{Sf}-(\gamma R_2-\lambda)\,T_{Rf}-(d+fR_2)\,T_{load}\right) \tag{20}$$

equation (15) can be written as

$$\left[\begin{array}{c}\dot{\omega}_S\\ \dot{\omega}_R\end{array}\right]=\tfrac{1}{a(c+eR_2)}\left(\left[\begin{array}{c}(de-cf)\,R_2\\ cf-de\end{array}\right]T_{load}\right.$$
$$+\left[\begin{array}{cc}-(e\tau+c\lambda)\,C_SR_2 & (e\lambda+c\gamma)\,C_RR_2\\ (e\tau+c\lambda)\,C_S & -(e\lambda+c\gamma)\,C_R\end{array}\right]\left[\begin{array}{c}\omega_S\\ \omega_R\end{array}\right]$$
$$+\left[\begin{array}{cc}(e\tau+c\lambda)\,R_2 & -(e\lambda+c\gamma)\,R_2\\ -(e\tau+c\lambda) & e\lambda+c\gamma\end{array}\right]\left[\begin{array}{c}T_{BS}\\ T_{BR}\end{array}\right]$$
$$\left.+\left[\begin{array}{cc}(e\tau+c\lambda)\,R_2 & -(e\lambda+c\gamma)\,R_2\\ -(e\tau+c\lambda) & e\lambda+c\gamma\end{array}\right]\left[\begin{array}{c}T_{Sf}\\ T_{Rf}\end{array}\right]\right) \tag{21}$$

which meets the requirement (18). The system (21) is not controllable, as the top row in its controllability matrix is $-R_2$ times the second row and hence is of rank 1. This is due to the geometrical constraint (16) between state components when $\omega_{C,out}$ is fixed. Substituting (16) into (21) gives

$$\dot{\omega}_S=\tfrac{1}{a(c+eR_2)}\Big(-\left[(e\tau+c\lambda)\,C_SR_2+(e\lambda+c\gamma)\,C_R\right]\omega_S$$
$$+(e\tau+c\lambda)\,R_2T_{BS}-(e\lambda+c\gamma)\,R_2T_{BR}$$
$$+(1+R_2)\,(e\lambda+c\gamma)\,C_R\omega_{C,out}+(de-cf)\,R_2T_{load}$$
$$+(e\tau+c\lambda)\,R_2T_{Sf}-(e\lambda+c\gamma)\,R_2T_{Rf}\Big) \tag{22}$$

For ease of notation, define

$$A_S:=\frac{(e\tau+c\lambda)\,C_SR_2+(e\lambda+c\gamma)\,C_R}{a\,(c+eR_2)},$$

$$B_{S1}:=\frac{(e\tau+c\lambda)\,R_2}{a\,(c+eR_2)},\quad B_{S2}:=\frac{(e\lambda+c\gamma)\,R_2}{a\,(c+eR_2)}$$

$$G_S:=\tfrac{1}{a(c+eR_2)}\Big((1+R_2)\,(e\lambda+c\gamma)\,C_R\omega_{C,out}$$
$$+(de-cf)\,R_2T_{load}+(e\tau+c\lambda)\,R_2T_{Sf}$$
$$-(e\lambda+c\gamma)\,R_2T_{Rf}\Big)$$

Thus (22) becomes

$$\dot{\omega}_S=-A_S\omega_S+B_{S1}T_{BS}-B_{S2}T_{BR}+G_S \tag{23}$$

with the initial and the terminal conditions from (33)

$$\omega_S\,(0)=\omega_S\,(GR_1)\,,\ \ \omega_S\,(t_f)=0 \tag{24}$$

for the upshift process. The reverse order for the initial and final conditions hold for the downshift process. Note that ω_R can either be computed from the geometrical constraint (16) or its differential equation which is dependent on the dynamics of ω_S.

In order to determine the admissible set for the control inputs, note that the brake of the sun is designed to be of the multi-plate clutch type. Thus, the relation between the normal applied force on the clutch plates and the resulting torque is:

$$T_{BS}=-\mu_PN_{BS}\,n(\frac{2}{3})(\frac{R_o^3-R_i^3}{R_o^2-R_i^2})sign(\omega_S);\quad N_{BS}\geq 0 \tag{25}$$

where μ_P is the coefficient of friction between clutch plates, N_{BS} is the normal brake applied to the plates and n is the number of the plates. The brake of the ring is designed to be of band brake type. So, the relation between the normal applied force at the end of the band and the resulting torque is:

$$\begin{cases}T_{BR}=-N_{BR}R_D(e^{\mu_D\theta}-1);\omega_R\geq 0, & N_{BR}\geq 0\\ T_{BR}=N_{BR}R_D(1-e^{-\mu_D\theta});\omega_R<0, & N_{BR}\geq 0\end{cases} \tag{26}$$

where N_{BR} is the force applied at the end of the band, R_D is the radius of the drum brake, μ_D is the coefficient of friction between band and drum and θ is the angle of wrap.

For the band brake the positive direction of rotation is considered as the energizing mode of the band brake and the negative direction as de-energizing one.

For changing the gear ratio smoothly and gradually, it is desired that the angular velocity of the sun gear goes to zero without any overshoot. At the beginning of the gear change, the speed of the sun gear is positive. By applying the normal brake of the sun, according to (25), the braking torque of the sun becomes negative. Furthermore, as the speed of the sun goes to zero gradually and the output speed is controlled to be constant, according to (5), the speed of the ring goes to a certain positive value. So, from (26) by applying a normal brake force, the brake torque of the ring becomes negative and it can be concluded that:

$$-\left|T_{BS}^{max}\right|\leq T_{BS}\leq 0,\quad -\left|T_{BR}^{max}\right|\leq T_{BR}\leq 0 \tag{27}$$

In order to design the Minimum Shifting-Time Controller, the Pontryagin Minimum Principle is employed and the Hamiltonian for the system (23) is formed as

$$\begin{array}{c}H\,(\omega_S,p,T_{BS},T_{BR})=\\ 1+p\,(-A_S\omega_S+B_{S1}T_{BS}-B_{S2}T_{BR}+G_S)\end{array} \tag{28}$$

The Minimum Principle states that along the optimal trajectory ω_S^o there exists p^o such that

$$\dot{p}^o=-\frac{\partial H\,(\omega_S,p^o,T_{BS},T_{BR})}{\partial\omega_S}=p^oA_S \tag{29}$$

and

$$H\,(\omega_S^o,p^o,T_{BS}^o,T_{BR}^o)\leq H\,(\omega_S^o,p^o,T_{BS},T_{BR}^o) \tag{30}$$

for all $-|T_{BS}^{max}| \le T_{BS} \le 0$, and

$$H\left(\omega_S^o, p^o, T_{BS}^o, T_{BR}^o\right) \le H\left(\omega_S^o, p^o, T_{BS}^o, T_{BR}\right) \quad (31)$$

for all $-|T_{BR}^{max}| \le T_{BR} \le 0$.

Since this is a fixed terminal value problem, the terminal value for the adjoint process is free. Solving (29) results in

$$p^o\left(t\right) = p^o\left(\tau\right) e^{A_s(\tau - t)} \quad (32)$$

with τ being the first time that the terminal condition in (24) occurs after t_s when the shifting begins. If $p^o\left(\tau\right) < 0$ then $p^o\left(t\right) < 0$ for all $t \in [t_s, \tau]$ and the Hamiltonian minimization (30) gives $T_{BS}^o = 0$ and (31) gives $T_{BR}^o = -|T_{BR}^{max}|$. This corresponds to the exponential growth of ω_S which is not desirable in the upshift process. If $p^o\left(\tau\right) > 0$ then $p^o\left(t\right) > 0$ for all $t \in [t_s, \tau]$ and the Hamiltonian minimization (30) gives $T_{BS}^o = -|T_{BS}^{max}|$ and (31) gives $T_{BR}^o = 0$. This case indeed gives the solution to the minimum time problem for upshift. Having established the optimal values for T_{BR} and T_{BS}, the motor torque is calculated from (20).

B. Feasible Controller

The ideal controller requires sudden engagement and release of the brakes which is beyond the actuators' limits. In addition, gear ratio variations for the optimal controller is not smooth at the beginning and the end of shifting. Thus, the constraint (18) is relaxed in order to provide additional flexibility for the controller to meet these requirements. It is shown, however, that the feasible controller is still able to provide nearly constant output velocity and output torque while the controller commands lie within physical constraints of the system. Taking the derivative of (5) with substitution of (13) results in

$$\begin{aligned}\dot{\omega}_{C,out} = {} & T_{BS}\left(\frac{\tau - \lambda R_2}{a(R_2+1)}\right) + T_{BR}\left(\frac{\gamma R_2 - \lambda}{a(R_2+1)}\right) \\ & + \omega_S C_S\left(\frac{\lambda R_2 - \tau}{a(R_2+1)}\right) + \omega_R C_R\left(\frac{\lambda - \gamma R_2}{a(R_2+1)}\right) \\ & + T_M\left(\frac{c + eR_2}{a(R_2+1)}\right) + T_{Load}\left(\frac{d + fR_2}{a(R_2+1)}\right) \\ & + T_{Sf}\left(\frac{\tau - \lambda R_2}{a(R_2+1)}\right) + T_{Rf}\left(\frac{\gamma R_2 - \lambda}{a(R_2+1)}\right)\end{aligned} \quad (33)$$

Since T_{load} ia a nonlinear function, input-output feedback linearization method is used by selecting T_M as follow:

$$\begin{aligned}T_M = {} & \frac{-1}{c + eR_2}\Big[(\tau - \lambda R_2)T_{BS} + (\gamma R_2 - \lambda)T_{BR} \\ & + (C_S\lambda R_2 - C_S\tau)\omega_S + (C_R\lambda - C_R\gamma R_2)\omega_R \\ & + (d + fR_2)T_{load} + (\tau - \lambda R_2)T_{Sf} + (\gamma R_2 - \lambda)T_{Rf} \\ & - K\frac{a(R_2+1)}{c + eR_2}(\omega_{C,out} - \omega_{C,out_D})\Big]\end{aligned} \quad (34)$$

where ω_{C,out_D} is the desired output speed of the transmission which is considered to be constant during the gear change. Thus (33) becomes:

$$\dot{\omega}_{C,out} = -K\left(\omega_{C,out} - \omega_{C,out_D}\right) \quad (35)$$

where K is the controller gain. This ensures the stability of the output speed and exponential convergence of the error $(\omega_{C,out} - \omega_{C,out_D})$ to zero without any overshoot. As mentioned before, the control inputs for this problem are T_M, T_{BS} and T_{BR}. From (34), the input control T_M is given as a function of T_{BS} and T_{BR}. Substituting T_M in the first equation of (13), it is concluded that:

$$\begin{aligned}\dot{\omega}_S = {} & \frac{1}{a(c + eR_2)}[-(e\tau + c\lambda)C_S R_2\omega_S + (e\lambda + c\gamma)C_R R_2\omega_R \\ & + (e\tau + c\lambda)R_2 T_{BS} - (e\lambda + c\gamma)R_2 T_{BR} \\ & + (de - cf)R_2 T_{load} + (e\tau + c\lambda)R_2 T_{Sf} \\ & - (e\lambda + c\gamma)R_2 T_{Rf} - cK(R_2+1)(\omega_{C,out} - \omega_{C,out_D})]\end{aligned} \quad (36)$$

Define:

$$\begin{aligned}\Delta := {} & \frac{1}{a(c + eR_2)}[-(e\tau + c\lambda)C_S R_2\omega_S + (e\lambda + c\gamma)C_R R_2\omega_R \\ & + (de - cf)R_2 T_{load} + (e\tau + c\lambda)R_2 T_{Sf} \\ & - (e\lambda + c\gamma)R_2 T_{Rf} - cK(R_2+1)(\omega_{C,out} - \omega_{C,out_D})]\end{aligned} \quad (37)$$

If $\Delta \ge 0$ take

$$\begin{aligned}T_{BS} = {} & \frac{-a(c + eR_2)}{e\tau + c\lambda}[\Delta - (e\lambda + c\gamma)R_2 T_{BR} \\ & + P(\omega_S - \omega_{S_D}) + \dot{\omega}_{S_D}]\end{aligned} \quad (38)$$

and if $\Delta < 0$ take

$$\begin{aligned}T_{BR} = {} & \frac{-a(c + eR_2)}{e\lambda + c\gamma}[\Delta + (e\tau + c\lambda)R_2 T_{BS} \\ & + P(\omega_S - \omega_{S_D}) + \dot{\omega}_{S_D}]\end{aligned} \quad (39)$$

where P is the controller gain and T_{BS} and T_{BR} are desired to follow their corresponding values in the ideal control strategy while satisfying the actuator limitations. This ensures the exponential convergence of ω_S to the desired trajectory in the form of $\dot{\omega}_S = -P(\omega_S - \omega_{S_D}) + \dot{\omega}_{S_D}$.

V. SIMULATION RESULTS

For the proposed transmission with parameters given in Table II, simulation results are provided for a sample upshift process. Fig.2-Fig.5 illustrate the results for the feasible controller. The gear changing process starts at $t = 20\,s$ and lasts about $1.5\,s$ for the feasible controllers. As it can be seen in Fig.3 sudden engagement and disengagement of the on-going an off-going brakes are replaced by feasible variations

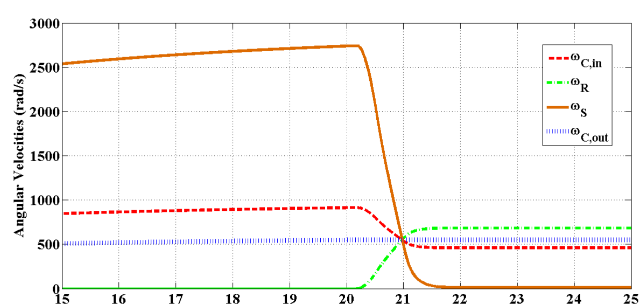

Fig. 2. Angular Velocities of the Input,Output, Ring and Sun

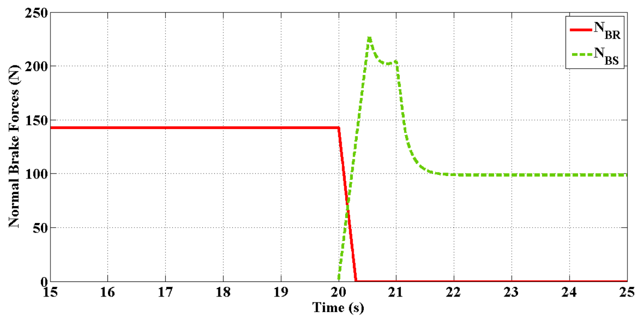

Fig. 3. Normal Braking Forces of the Ring and Sun

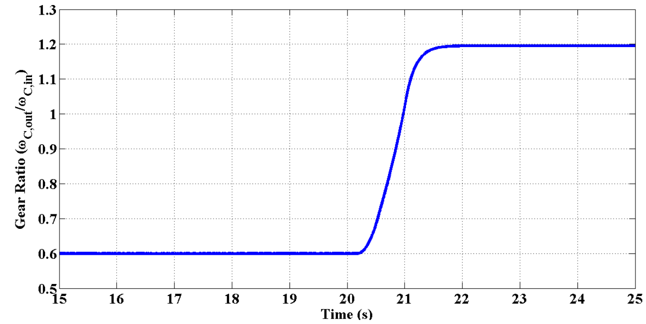

Fig. 5. Gear Ratio

of the normal brake forces (normal brake forces can be calculated from (25) and (26)). As it can be observed in Fig.4 this corresponds to lower peak values for the motor torque in comparison with the ideal controller. Fig.2 demonstrates the input and output speed of the transmission as well as speeds of the ring and the sun gears. It can be seen that the oscillation of the output speed and the output torque during the gear change is almost negligible. Fig.5 demonstrates the variation of the gear ratio during the gear changing procedure.

TABLE II
PARAMETERS OF THE DYNAMIC SYSTEM

$r_{R,in}(m)$	6e-2	$I_R(Kg.m^2)$	9e-3
$r_{R,out}(m)$	6e-2	$I_S(Kg.m^2)$	1.5e-3
$r_{S,in}(m)$	3e-2	$I_{C,in}(Kg.m^2)$	1.4e-3
$r_{S,out}(m)$	15e-3	$I_{C,out}(Kg.m^2)$	0.1
$r_{P,in}(m)$	15e-3	$I_{P,in}(Kg.m^2)$	6.08e-6
$r_{P,out}(m)$	22.5e-3	$I_{P,out}(Kg.m^2)$	3.12e-5
$C_R(N.m.s/rad)$	0.001	$m_{P,in}(Kg)$	0.0512
$C_S(N.m.s/rad)$	0.001	$m_{P,out}(Kg)$	0.12113
$T_{Rf}(Nm)$	0.05	$T_{Sf}(Nm)$	0.05
μ_R, μ_S	0.15	n	4
$R_o(m)$	0.09	$R_i(m)$	0.08
$R_D(m)$	0.1	$\theta(rad)$	$3\pi/4$

VI. CONCLUSION

In this paper, an analytical dynamic model of the proposed two-speed automated transmission is presented and validated by MATLAB/*Simulink*®. An ideal controller is designed in order to keep the output speed and torque constant during gear changes. In order to cope with the actuator limits, a feasible controller is developed by relaxing the constraints in design of the ideal controller. Simulation results show that the oscillation of the output torque and output speed, during the gear change, remains less than 5%. The proposed transmission with the controller algorithm developed in this paper provides seamless gearshifts, while the two embedded brake systems help in the synchronization of the speed of the motor with the driveline which reduces the shifting time.

Future work will be validating the proposed modeling and control algorithm on the real testbed and improve the controller to consider all the possible uncertainties and disturbances.

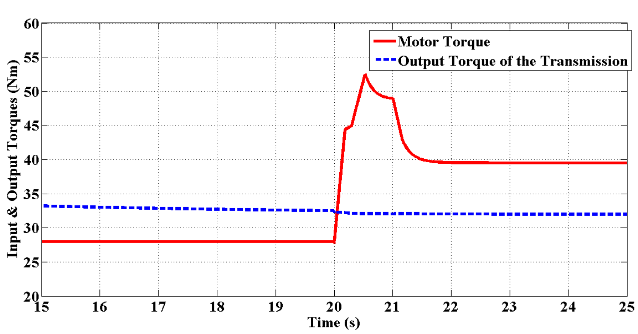

Fig. 4. Motor Torque and Output Torque

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