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# Distributed Adaptive Consensus with Obstacle and Collision Avoidance for Networks of Heterogeneous Multi-Agent Systems

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*Abstract*— **This paper presents a distributed adaptive control strategy for multi-agent systems with heterogeneous time-varying nonlinear dynamics, integrating obstacle and collision avoidance mechanisms. We propose an adaptive control strategy designed to ensure leader-following formation consensus while effectively managing collision and obstacle avoidance using potential functions. By integrating neural network (NN) based estimation and adaptive tuning laws, the proposed strategy ensures consensus, safety and stability in leader-following formations under fixed topologies. The NNs enhance the adaptability of the proposed architecture, enabling agents to learn and adjust to environmental and system changes.**

## I. Introduction

Cooperative control of multi-agent systems (MAS) is crucial for UAV coordination, autonomous vehicle platooning, and smart grids. In these systems, agents interact only with a limited number of other agents, creating a graph-theoretic structure that complicates achieving global consensus [1], [2]. The challenge intensifies in heterogeneous MAS with high-order nonlinear dynamics and external disturbances [3], further compounded by the need for real-time adaptation to unknown parameters through limited interactions.

Despite advancements in the field, significant gaps remain in addressing adaptive control for leader-following consensus with obstacle and collision avoidance in nonlinear systems. Adaptive tracking control methods [1], [4] and consensus approaches for switching topologies [5] often neglect physical constraints like collision avoidance. While adaptive tracking methods [4] are effective for dynamic topologies and active leaders, they fail to address heterogeneous agent dynamics and fixed topology networks. Research on local interaction rules [6] and formation control conditions [7] offers insights into geometric constraints but overlooks the interplay of consensus and safety constraints. Stability analyses under time-dependent links [8] highlight network challenges but ignore real-time nonlinear estimation, and studies on switching topologies [9] partially address physical constraints and heterogeneous dynamics.

**Contributions:** While consensus algorithms and collision avoidance mechanisms have been extensively studied, their integration with adaptive control in heterogeneous systems remains limited. This paper bridges the gap by proposing a unified control and parameter update law. It employs NN estimation [1] to integrate adaptive distributed consensus, time-varying nonlinear dynamics, and obstacle/collision avoidance for heterogeneous MAS. The approach ensures leader-formation consensus in dynamic and uncertain environments. A key challenge is accurately estimating diverse agent dynamics, often partially unknown or influenced by external disturbances. Traditional model-based methods struggle with real-time adaptation, but NNs, embedded in the proposed distributed architecture, serve as adaptive estimators, learning agent dynamics and environmental variations online. This enhances robustness, enabling adaptive consensus and collision-free trajectories through dynamic control law updates. The proposed strategy uniquely integrates these solutions, offering a robust framework for heterogeneous MAS with fixed topologies, collision/obstacle avoidance and external disturbances, addressing prior research limitations by unifying all critical features. The rest of the paper is organized as follows:

Section II defines MAS dynamics and the leader-following framework. Section III presents an adaptive control protocol with neural network-based learning for nonlinear dynamics and disturbances. Section IV proves control law stability and convergence. Section V demonstrates the strategy via a numerical leader-follower example. Section VI summarizes contributions and potential future work.

**Notations:** Absolute value is $|\cdot|$; vector Euclidean norm $\|\cdot\|$; matrix Frobenius norm $\|\cdot\|_F$; matrix trace $\mathrm{tr}\{\cdot\}$; singular values $\sigma(\cdot)$, with $\bar{\sigma}(\cdot)$ and $\underline{\sigma}(\cdot)$ as maximum and minimum singular values, respectively. Additional notations are introduced in the text as needed.

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## II. Problem Formulation

We define the set of follower agents as $\mathcal{N} = \{1, ..., N\}$. The dynamics of the $i^{\text{th}}$ follower agent is described using the nonlinear Brunovsky form as follows:

$$\begin{aligned} \dot{x}_i^1 &= x_i^2 \\ \dot{x}_i^2 &= x_i^3 \\ &\vdots \\ \dot{x}_i^n &= f_i(x_i) + u_i + w_i, \end{aligned} \tag{1}$$

where $k = \{1, 2, \ldots, n\}$, $x_i^k \in \mathbb{R}$ is the $k$-th state of agent $i$, and $x_i = [x_i^1, x_i^2, ..., x_i^n] \in \mathbb{R}^n$ denoting its state vector, $u_i \in \mathbb{R}$ represents the control input of agent $i$, and $w_i \in \mathbb{R}$ denotes the bounded unknown time-varying disturbance affecting agent $i$.

The unknown functions $f_i(x) : \mathbb{R}^n \to \mathbb{R}$ are locally Lipschitz in $x$ with $f_i(0) = 0$, for all $i \in \mathcal{N}$. Although the governing dynamics (1) of the agents are decoupled, their interactions become coupled through collision and obstacle avoidance, as discussed further in this section.

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We utilize the Kronecker product [10] to collectively represent (1) in the following global form:

$$\begin{aligned} \dot{x}^1 &= x^2 \\ &\vdots \\ \dot{x}^n &= f(x) + u + w, \end{aligned} \tag{2}$$

where $x^k = [x_1^k, x_2^k, \dots, x_N^k]^\top \in \mathbb{R}^N$ denotes the state vector of the follower agents, $u = [u_1, u_2, \dots, u_N]^\top \in \mathbb{R}^N$ denotes their control inputs, and $w = [w_1, w_2, \dots, w_N]^\top \in \mathbb{R}^N$ denotes unknown bounded disturbances. The unknown function $f(x) = [f_1(x_1), f_2(x_2), \dots, f_N(x_N)]^\top \in \mathbb{R}^{nN}$ represents the dynamics of the entire $N$ follower agents.

In this paper, we consider a leader-follower scenario where the leader agent, denoted by subscript 0, generates a reference trajectory for follower agents, but this reference trajectory is a priori unknown to all follower agents, and the only information about the reference trajectory available to follower agents is the leader's state vector $x_0 \equiv x_0(t) \in \mathbb{R}^n$ at the current time $t \in [t_0, \infty)$.

The time-varying dynamics of the leader agent is:

$$\begin{aligned} \dot{x}_0^1(t) &= x_0^2(t) \\ &\vdots \\ \dot{x}_0^n(t) &= f_0(x_0, t), \end{aligned} \tag{3}$$

where $x_0^k \in \mathbb{R}$ is the $k$-th component of the leader agent's state, and $x_0 = [x_0^1, x_0^2, ..., x_0^n]$ is its state vector.

The unknown function $f_0(x_0, t) : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ in the leader dynamics is considered to be piecewise continous in $t$ and locally Lipschitz in $x_0$, with $f_i(0, t) = 0$ for all $t \geq 0$, and all $x_0 \in \mathbb{R}^n$.

In the consensus problem, it is desired (see, e.g., [11]) that

$$\lim_{t \to \infty} [(x_i^k - \psi_i) - (x_0^k - \psi_0)] = 0, \tag{4}$$

$$\lim_{t \to \infty} [(x_i^k - \psi_i) - (x_j^k - \psi_j)] = 0, \tag{5}$$

for each $i \in \mathcal{N}$ and for all $j \in \mathcal{N}, j \neq i$, where $\psi_i$, $\psi_j$ and $\psi_0$ denote the desired offsets for agents $i$, $j$ and 0, respectively. However, such a strong level of connectivity which requires communication among all agents is often not feasible in networked control systems due to practical constraints such as limited bandwidth, packet loss, and delays (e.g., see [12]).

This paper proposes a decentralized consensus policy where each agent computes its input $u_i$ using its own state $x_i$ and the states of a limited subset of agents, which may or may not include the leader agent. Potential functions, extending [10], are employed to ensure collision and obstacle avoidance, guiding agents to safely navigate their environment.

For collision avoidance between agent $i$ and agent $j$ positions, we define $m_{ij}$ as follows:

$$m_{ij} := \begin{cases} 0, & \|x_i^1 - x_j^1\| \geq \psi_{ij} \\ \frac{\chi}{\left|\left|x_i^1 - x_j^1\right|\right|}. & \|x_i^1 - x_j^1\| < \psi_{ij} \end{cases} \tag{6}$$

Similarly, for collision avoidance between agent $i$ and the leader agent 0, we define $m_{i0}$ as follows:

$$m_{i0} := \begin{cases} 0, & \|x_i^1 - x_0^1\| \geq \psi_{i0} \\ \frac{\chi}{\left|\left|x_i^1 - x_0^1\right|\right|}. & \|x_i^1 - x_0^1\| < \psi_{i0}. \end{cases} \tag{7}$$

Moreover, for obstacle avoidance between agent $i$ and obstacle $\Omega$, we define $m_{i\bar{b}}$ as follows:

$$m_{i\bar{b}} := \begin{cases} 0, & R < ||x_i^1 - \Omega|| \\ \left[\frac{R^2 - ||x_i^1 - \Omega||^2}{\left|\left|x_i^1 - \Omega\right|\right|^2 - \partial^2}\right]^2. & \partial < ||x_i^1 - \Omega\| \leq R. \end{cases} \tag{8}$$

The formulation for obstacle avoidance between the leader agent 0 and obstacle $\Omega_{\bar{b}}$ is the same, with $x_i^1$ replaced by $x_0^1$. In these expressions, $\chi$ is a positive scalar adjusting the repulsive force strength, $\psi_{ij}$ is the desired separation between agents $i$ and $j$, $\psi_{i0}$ is the desired safety separation between agent $i$ and the leader, $R$ is the obstacle detection radius, and $\partial$ is the desired safety obstacle radius.

We define a communication topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the set of agents, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. A graph is undirected if $(v_i, v_j) \in \mathcal{E} \iff (v_j, v_i) \in \mathcal{E}$; otherwise, it is directed if $(v_i, v_j) \in \mathcal{E}$ but $(v_j, v_i) \notin \mathcal{E}$. The weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ models the interaction strength between agents, where:

$$a_{ij} := \begin{cases} w_{ij}, & \text{if } (i,j) \in \mathcal{E} \\ 0. & \text{otherwise} \end{cases}$$

Here, $w_{ij}$ represents the positive edge weight between agents $i$ and $j$. The in-degree matrix $D = \operatorname{diag}\{d_i\} \in \mathbb{R}^{N \times N}$ is diagonal, where $d_i = \sum_{j=1}^N a_{ij}$ is the sum of the edge weights connected to node $i$. The Laplacian matrix $L = D - A$ is positive semi-definite, with one zero eigenvalue if the graph is connected. With a leader agent, the augmented graph is $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \{v_0, v_1, \dots, v_N\}$, and $B = \operatorname{diag}\{b_i^0\} \in \mathbb{R}^{N \times N}$ denotes the leader's interaction, where:

$$b_i^0 := \begin{cases} w_{i0}, & \text{if } (i,0) \in \bar{\mathcal{E}} \\ 0. & \text{otherwise} \end{cases}$$

Here, $w_{i0}$ represents the positive weight between agent $i$ and the leader 0. The augmented graph $\bar{\mathcal{G}}$ includes a spanning tree rooted at the leader 0, facilitating agent coordination through state communication.

*Assumption 2.1:* The adjacency matrix $A$ meets the following conditions:

(a) The augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with the leader as the root; this ensures that no clusters of agents are isolated from the leader. In other words, whenever $(i, 0) \notin \bar{\mathcal{E}}$, there exists a sequence of nonzero elements of $A$ of the form $a_{ii_2}, a_{i_2 i_3}, \cdots, a_{i_{l-1} i'}$, for some $(i', 0) \in \bar{\mathcal{E}}$, with the sequence length $l$ being a finite integer.

(b) If $\|x_i - x_j\| \leq \Psi$ with $\Psi \in \mathbb{R}_{>0}$ denoting a proximity threshold, then $a_{ij} = 1$; this ensures that agents within a $\Psi$-range are connected and can exchange information.

*Assumption 2.2:*

1) The initial states of all follower agents and the leader

agent are bounded, i.e., $||x_i(t_0)|| \leq X_n$ for all $i \in \mathcal{N}$, and $||x_0(t_0)|| \leq X_{n0}$.
2) There exist continuous functions $f_n(\cdot)$ and $f_{n0}(\cdot)$ such that $|f_i(x)| \leq |f_n(x)|$ for all $i \in \mathcal{N}$, and $|f_0(x_0,t)| \leq |f_{n0}(x_0)|$ for all $x$ within the compact sets $\varsigma_f = \{x \mid \|x\| \leq X_n\}$ and $\varsigma_0 = \{x_0 \mid \|x_0\| \leq X_{n0}\}$.
3) The input $u(t)$ is bounded within any finite time interval, i.e., $||u(t)|| \leq u_n$ for all $t \in [t_0, T]$, $T \geq t_0$.
4) The unknown disturbance $w_i$ for each agent is bounded, i.e., $||w(t)|| \leq w_n$ for all $t \in [t_0, \infty)$.

## III. Methodology

### A. Neural Network Learning Problem

In order to account for uncertainties in the dynamics, we use a distributed two-layer linear-in-parameters (LIP) neural network [13], enabling each agent to locally update its model. In this framework, each agent independently uses a neural network to model the nonlinear dynamics of itself and the agents connected to it in real-time. This also yields to a reduced number of required neurons in comparison to centralized approaches. Accordingly, we model the functions as:

$$f_i(x_i) = \theta_i^\top \phi_i(x_i) + \varepsilon_i, \tag{9}$$

$$f_0(x_0,t) = \theta_0^\top \phi_0(x_0,t) + \varepsilon_0, \tag{10}$$

$$w_i(t) = \theta_{iw}^\top \phi_{iw}(t) + \varepsilon_{iw}. \tag{11}$$

where $\phi_i(x_i)$, $\phi_0(x_0,t)$, and $\phi_{iw}(t)$ are fixed basis functions, with the corresponding weight vectors $\theta_i$, $\theta_0$, and $\theta_{iw}$, updated in real-time as new data is received. The approximation errors are $\varepsilon_i$, $\varepsilon_0$, and $\varepsilon_{iw}$.

The neural network approximations for $f_i(x_i)$, $f_0(x_0,t)$, and $w_i(t)$ are therefore $\hat{f}_i(x_i) = \hat{\theta}_i^\top \phi_i(x_i)$, $\hat{f}_0(x_0,t) = \hat{\theta}_0^\top \phi_0(x_0,t)$, $\hat{w}_i(t) = \hat{\theta}_{iw}^\top \phi_{iw}(t)$, where the NN weights $\hat{\theta}_i$, $\hat{\theta}_0$, and $\hat{\theta}_{iw}$ are computed locally. For the brevity of notation, the basis functions and errors are denoted by $\theta = \mathrm{diag}(\theta_1,\ldots,\theta_N)$, $\phi(x) = [\phi_1^\top(x_1),\ldots,\phi_N^\top(x_N)]^\top$, $\varepsilon = [\varepsilon_1,\ldots,\varepsilon_N]^\top$. Other parameters follow a similar pattern. The global nonlinearities are expressed as:

$$f(x) = \theta^\top \phi(x) + \varepsilon, \tag{12}$$

$$f_0(x,t) = \theta_0^\top \phi_0(x,t) + \varepsilon_0, \tag{13}$$

$$w(t) = \theta_w^\top \phi_w(t) + \varepsilon_w, \tag{14}$$

with approximations:

$$\hat{f}(x) = \hat{\theta}^\top \phi(x), \tag{15}$$

$$\hat{f}_0(x,t) = \hat{\theta}_0^\top \phi_0(x,t), \tag{16}$$

$$\hat{w}(t) = \hat{\theta}_w^\top \phi_w(t). \tag{17}$$

The NN weight errors are: $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{\theta}_0 = \theta_0 - \hat{\theta}_0$, $\tilde{\theta}_w = \theta_w - \hat{\theta}_w$.

The $k^{\text{th}}$ order tracking error is defined as $\delta_i^k = (x_i^k - \psi_i) - (x_0^k - \psi_0)$, and globally as

$$\delta^k = [\delta_1^k, \delta_2^k, \ldots, \delta_N^k]^\top \in \mathbb{R}^N. \tag{18}$$

*Definition 3.1:* The tracking error $\delta^k$ is *cooperatively uniformly ultimately bounded* (CUUB) [1] if there exists a compact set $\varsigma^k \subset \mathbb{R}^N$ with $\{0\} \subset \varsigma^k$ such that for any initial condition $\delta^k(t_0) \in \varsigma^k$, there exists $B^k \in \mathbb{R}_{>0}$ and $T_k \in \mathbb{R}_{>0}$ which yield $||\delta^k(t)|| \leq B^k$ for all $t \geq t_0 + T_k$.

*Assumption 3.1:* The basis functions $\phi_i(x_i)$, $\phi_0(x_0,t)$, $\phi_{iw}(t)$, the NN weights $\theta_i$, $\theta_0$, $\theta_{iw}$, and the approximation errors $\varepsilon$, $\varepsilon_0$, $\varepsilon_w$ are bounded by specified positive constants.

Let $\phi_{in} = \max_{x_i \in \varsigma} \|\phi_i(x_i)\|$, $\phi_{0n} = \max_{x_0 \in \varsigma, t \geq 0} \|\phi_0(x_0,t)\|$, and $\phi_{iwn} = \max_{t \geq 0} \|\phi_{iw}(t)\|$. Based on Assumption 3.1, there exist positive numbers $\Phi_n$, $\Theta_n$, and $\varepsilon_n$; $\Phi_{n0}$, $\Theta_{n0}$, and $\varepsilon_{n0}$; $\Phi_{nw}$, $\Theta_{nw}$, and $\varepsilon_{nw}$, such that: $\|\phi_i(x_i)\| \leq \Phi_n$, $\|\phi_0(x_0,t)\| \leq \Phi_{n0}$, $\|\phi_{iw}(t)\| \leq \Phi_{nw}$, $\|\theta_i\|_F \leq \Theta_n$, $\|\theta_0\|_F \leq \Theta_{n0}$, $\|\theta_{iw}\|_F \leq \Theta_{nw}$, $\|\varepsilon\| \leq \varepsilon_n$, $\|\varepsilon_0\| \leq \varepsilon_{n0}$, $\|\varepsilon_w\| \leq \varepsilon_{nw}$.

### B. Local Synchronization Error

Given the limited information available to the $i^{\text{th}}$ agent, we define its $k^{\text{th}}$ order weighted synchronization error as:

$$e_i^k = -\nu_1 \sum_{j=1}^N a_{ij} \Big[ (x_i^k - \psi_i) - (x_j^k - \psi_j) \Big] - \nu_2 b_i^0 \Big[ (x_i^k - \psi_i) - (x_0^k - \psi_0) \Big], \tag{19}$$

where the parameters $\nu_1$ and $\nu_2$, are positive scalar gains that adjust the influence between agents and the leader, providing flexibility in varying degrees of influence across the network.

Using the simplifying notations: $\bar{x}_i^k := x_i^k - \psi_i$, $\bar{x}_j^k := x_j^k - \psi_j$ and $\bar{x}_0^k := x_0^k - \psi_0$, we rewrite equation (19) as:

$$e_i^k = -\nu_1 \sum_{j=1}^N a_{ij} \Big[ \bar{x}_i^k - \bar{x}_j^k \Big] - \nu_2 b_i^0 \Big[ \bar{x}_i^k - \bar{x}_0^k \Big]. \tag{20}$$

Here, $-\nu_1 \sum_{j=1}^N a_{ij}(\bar{x}_i^k - \bar{x}_j^k)$ ensures consensus among agents by penalizing the difference between the state of agent $i$ and its neighbors, while $-\nu_2 b_i^0 (\bar{x}_i^k - \bar{x}_0^k)$ couples the agent's state to the leader state by penalizing their difference. Defining, $e^k = [e_1^k, e_2^k, e_3^k, \ldots, e_N^k]^\top$, $\underline{\bar{x}}_0^k = [\bar{x}_0^k, \ldots, \bar{x}_0^k]^\top$, $\bar{x}^k = [\bar{x}_1^k, \ldots, \bar{x}_N^k]^\top$, we reformulate (20) in global form as:

$$e^k = -(\nu_1 L + \nu_2 B)(\bar{x}^k - \underline{\bar{x}}_0^k). \tag{21}$$

*Lemma 1:* Under Assumption 2.1, the matrix $\nu_1 L + \nu_2 B$ is nonsingular.

*Proof:* Due to page limitations, the proof is not presented here. See [14] for the detailed proof. ■

### C. Local Error Dynamics

The error dynamics are derived by differentiating the synchronization error from (20):

$$\begin{aligned} \dot{e}^k &= e^{k+1}, \quad k = 1, ..., n-1 \\ \dot{e}^k &= -(\nu_1 L + \nu_2 B)(\dot{\bar{x}}^n - \dot{\underline{\bar{x}}}_0^n), \quad k = n \end{aligned} \tag{22}$$

which, in the expanded form, is written as:

$$\begin{aligned} \dot{e}^1 &= e^2 = -(\nu_1 L + \nu_2 B)(\bar{x}^2 - \underline{\bar{x}}_0^2), \\ \dot{e}^2 &= e^3 = -(\nu_1 L + \nu_2 B)(\bar{x}^3 - \underline{\bar{x}}_0^3), \\ &\vdots \\ \dot{e}^n &= -(\nu_1 L + \nu_2 B)(f(\bar{x}) + u + w - f_0(\bar{x}_0, t)). \end{aligned} \tag{23}$$

### D. Weighted Stability Error

The weighted stability error $r_i$ is defined as a linear combination of error terms for each follower agent $i$:

$$r_i = \lambda_1 e_i^1 + \lambda_2 e_i^2 + \cdots + \lambda_{n-1} e_i^{n-1} + e_i^n, \tag{24}$$

where the design parameters $\lambda_j$ (for $j = 1, \ldots, n-1$) are chosen such that the characteristic polynomial:

$$s^{n-1} + \lambda_{n-1} s^{n-2} + \cdots + \lambda_1, \tag{25}$$

is Hurwitz. This ensures that all roots have negative real parts, leading to the stability of the associated linear system. The selection of $\lambda_j$ can alternatively be made by setting $s^{n-1} + \lambda_{n-1} s^{n-2} + \cdots + \lambda_1 = \prod_{j=1}^{n-1}(s - \xi_j)$., and selecting $\xi_j$'s to be positive real numbers. The global form of the representation of the weighted stability error is:

$$r = \lambda_1 e^1 + \lambda_2 e^2 + \cdots + \lambda_{n-1} e^{n-1} + e^n. \tag{26}$$

By differentiating (26) with respect to time, the weighted stability error dynamics are:

$$\begin{aligned} \dot{r} &= \lambda_1 \dot{e}^1 + \lambda_2 \dot{e}^2 + \cdots + \lambda_{n-1} \dot{e}^{n-1} + \dot{e}^n \\ &= \lambda_1 e^2 + \lambda_2 e^3 + \cdots + \lambda_{n-1} e^n - (\nu_1 L + \nu_2 B)(\dot{\bar{x}}^n - \dot{\bar{x}}_0^n), \end{aligned} \tag{27}$$

which can be written as

$$\dot{r} = \rho - (\nu_1 L + \nu_2 B)(f(x) + u + w - f_0(x_0, t)), \tag{28}$$

where $\rho = \lambda_1 e^2 + \lambda_2 e^3 + \lambda_3 e^4 + \cdots + \lambda_{n-1} e^n = E_2 \bar{\lambda}$ with $\bar{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_{n-1}]^\top$ and $E_2 = [e^2, e^3, ..., e^n]^\top$.

Representing $E_2$ in matrix form yield:

$$E_2 = E_1 \triangle^\top + r l^\top,$$

where $E_1 = [e^1, e^2, ..., e^{n-1}] \in \mathbb{R}^{N \times (n-1)}$, $E_2 = \dot{E}_1 = [e^2, e^3, ..., e^n] \in \mathbb{R}^{N \times (n-1)}$, $l = [0, 0, ..., 0, 1]^\top \in \mathbb{R}^{(n-1)}$,

$$\Delta = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\lambda_1 & -\lambda_2 & -\lambda_3 & \cdots & -\lambda_{n-1} \end{bmatrix} \in \mathbb{R}^{(n-1)\times(n-1)}$$

As stated in [1], since $\triangle$ is Hurwitz, there exists a positive-definite matrix $P_1$ such that the following condition holds for any positive number $\beta$:

$$\triangle^\top P_1 + P_1 \triangle = -\beta I, \tag{29}$$

where $I \in \mathbb{R}^{(n-1)\times(n-1)}$.

*Lemma 2:* If $r_i(t)$ is ultimately bounded, then $e_i(t)$ is ultimately bounded.

*Proof:* See [1, Lemma 10.3]. ■

*Lemma 3 (Graph Lyapunov Equation):* Define

$$q \equiv [q_1, \cdots, q_N]^\top := (\nu_1 L + \nu_2 B)^{-1} \underline{1}, \tag{30}$$

$$P \equiv \text{diag}\{p_i\}_{i \in \mathcal{N}} := \text{diag}\{1/q_i\}_{i \in \mathcal{N}}, \tag{31}$$

$$Q := P(\nu_1 L + \nu_2 B) + (\nu_1 L + \nu_2 B)^\top P, \tag{32}$$

where $\underline{1} = [1, ..., 1]^\top \in \mathbb{R}^N$. then the matrices $P$ and $Q$ are positive definite.

*Proof:* The proof is similar to [1, Lemma 10.1] by noting that $\nu_1 L + \nu_2 B$ is nonsingular by Lemma 1. ■

## IV. Main Result

*Theorem 1:* Let $\kappa_i > 0$, $\kappa_{iw} > 0$, and $\kappa_0 > 0$, then for the multi-agent system with the follower dynamics (1) and leader dynamics (3), and under Assumptions 2.1, 2.2 and 3.1, the NN tuning laws

$$\dot{\hat{\theta}}_i = -F_i \Big[ \phi_i r_i p_i (d_i + b_i^0) + \kappa_i \hat{\theta}_i \Big], \tag{33}$$

$$\dot{\hat{\theta}}_0 = F_{i0} \Big[ \phi_0 r_i p_i (d_i + b_i^0) - \kappa_0 \hat{\theta}_0 \Big], \tag{34}$$

$$\dot{\hat{\theta}}_{iw} = -F_{iw} \Big[ \phi_{iw} r_i p_i (d_i + b_i^0) + \kappa_{iw} \hat{\theta}_{iw} \Big], \tag{35}$$

together with the distributed control law

$$\begin{aligned} u_i = \frac{\rho}{d_i + b_i^0} - \hat{\theta}_i^\top \phi_i - \hat{\theta}_{iw}^\top \phi_{iw} + \hat{\theta}_0^\top \phi_0 + r_i - c_i E_{i0}^\top \\ - \Gamma_b^0 \sum_{b=1}^{\xi} m_{i\bar{b}} - \Gamma_{ij}^1 \sum_{j=1}^{N} m_{ij} - \Gamma_{i0}^2 \sum_{j=1}^{N} m_{i0}, \end{aligned} \tag{36}$$

guarantee synchronization, stability, and collision/obstacle avoidance, i.e.,:

1) Synchronization: The tracking errors, $\delta^k$ in (18), for all follower agents, $i \in \mathcal{N}$ are cooperatively uniformly ultimately bounded (CUUB).
2) Stability: The origin is globally stable for all $k^{\text{th}}$ order weighted synchronization errors (19), i.e., $e_i^k$ for all $k \in \{1, \cdots, n\}$, $i \in \{1, \cdots, N\}$, the weighted stability error (26), and the NN weight errors $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{\theta}_0 = \theta_0 - \hat{\theta}_0$, $\tilde{\theta}_w = \theta_w - \hat{\theta}_w$.
3) Collision/Obstacle Avoidance: Agents avoid collisions and obstacles while maintaining synchronization and stability, i.e., $m_{ij}$ in (6), $m_{i0}$ in (7), and $m_{i\bar{b}}$ in (8) remain bounded for all $t \geq 0$.

*Proof:* Due to space limitations, the detailed proof of theorem 1 is provided in the arXiv version [14]. ■

## V. Numerical Example

We consider a leader-follower multi-agent formation with one leader and five followers, where each agent has distinct nonlinear dynamics influenced by external disturbances. Figure 1 shows the information flow structure of the multi-agent system. The safety distance between agents is given as $\psi_{ij} = \psi_{i0}$ = 2. The leader's dynamics are given by:

**Leader**:

$$\dot{s_0} = v_0 \tag{37}$$

$$\begin{aligned} \dot{v_0} = -3v_0 + 1 - g\sin(\alpha(s_0)) - \frac{0.4 v_0^2}{m_0} \\ + \frac{3\sin(2t) + 6\cos(2t)}{m_0} - \frac{(s_0 + v_0 - 1)^2}{3m_0}(s_0 + 4v_0 - 1) \end{aligned}$$

Each follower $i$ ($i = 1, \ldots, 5$) has second-order nonlinear dynamics:

**Agent 1**:

$$\dot{s_1} = v_1 \tag{38}$$

$$\dot{v_1} = \frac{v_1 \sin(s_1)}{m_1} + \cos^2(v_1) - \frac{0.47 v_1^2}{m_1} - g\sin(\alpha(s_1)) + \frac{u_1}{m_1}$$

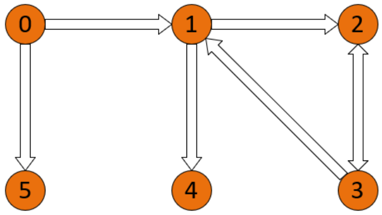


Fig. 1: The Considered Fixed Topology of the augmented graph $\bar{\mathcal{G}}$ in Section V.

$$+\frac{w_1}{m_1}$$

**Agent 2**:

$$\dot{s_2} = v_2 \tag{39}$$
$$\dot{v_2} = \frac{-(s_2)^2 v_2}{m_2} + \cos^2(v_2) - \frac{0.52 v_2^2}{m_2} - g\sin(\alpha(s_2)) + \frac{u_2}{m_2} + \frac{w_2}{m_2}$$

**Agent 3**:

$$\dot{s_3} = v_3 \tag{40}$$
$$\dot{v_3} = \frac{-(s_3)^2 v_3}{m_3} + \sin^2(v_3) - \frac{0.57 v_3^2}{m_3} - g\sin(\alpha(s_3)) + \frac{u_3}{m_3} + \frac{w_3}{m_3}$$

**Agent 4**:

$$\dot{s_4} = v_4 \tag{41}$$
$$\dot{v_4} = \frac{-3(s_4+v_4-1)^2(s_4+v_4-1)}{m_4} - v_4 + 0.5\sin(2t) + \cos(2t) - \frac{0.65 v_4^2}{m_4} - g\sin(\alpha(s_4)) + \frac{u_4}{m_4} + \frac{w_4}{m_4}$$

**Agent 5**:

$$\dot{s_5} = v_5 \tag{42}$$
$$\dot{v_5} = \cos(s_5) - \frac{0.74 v_5^2}{m_5} - g\sin(\alpha(s_5)) + \frac{u_5}{m_5} + \frac{w_5}{m_5}$$

Figures 2 to 6 analyze the proposed control strategy's effectiveness in ensuring synchronization, stability, and disturbance handling. Initially, agents begin below the leader, with Figure 2 illustrating cohesive formation maintenance through regulated inter-agent distances. Over time, agents' velocities converge to the leader's (Figure 3), dynamically synchronizing trajectories. Minimal relative position errors (Figure 4) indicate precise positioning and robustness against disturbances. Velocity errors (Figure 5) show transient variability, particularly for Agent 4, but stabilize over time, demonstrating adaptability and synchronization. Control inputs (Figure 6) reveal varied demands across agents, reflecting individual dynamic conditions—e.g., Agent 1 requires higher inputs, while Agent 5 shows increasing effort over time. These results validate the framework's robustness, with neural network-driven updates enabling consensus and safety in dynamic environments. These results validate the framework's robustness, with neural network-driven updates enabling consensus and safety in dynamic environments.

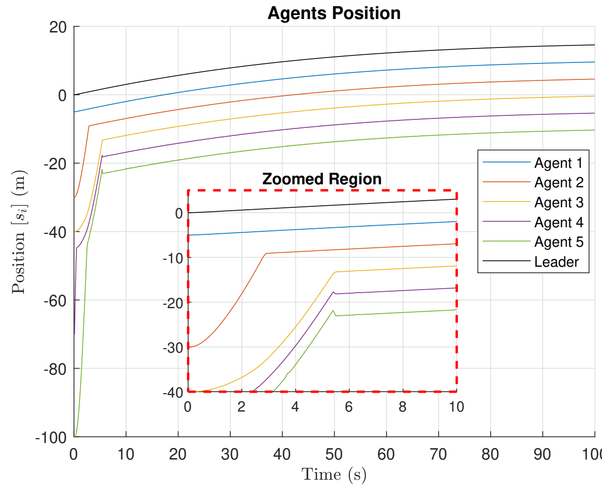


Fig. 2: Agents Positions $s_i$, for $i \in \{1,2,3,4,5\}$ and the leader 0.

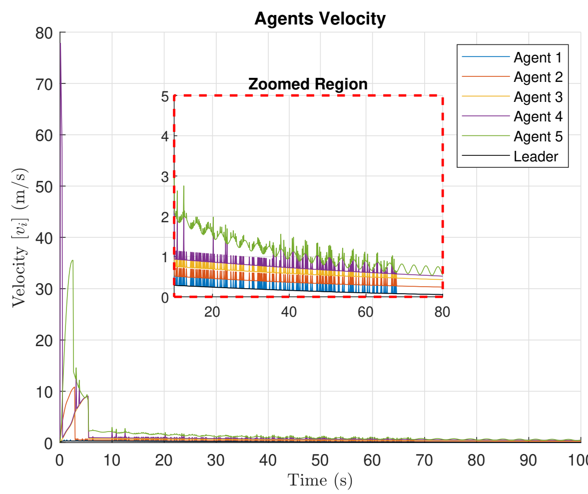


Fig. 3: Agents Velocities $v_i$, for $i \in \{1,2,3,4,5\}$.


Fig. 4: Relative Position Error, $E_{i0}$, for the first state, $k = 1$, between Agents and Leader.

TABLE I: Parameters for Leader and Followers

| **Parameter** | **Leader** ($i = 0$) | **Agent 1** ($i = 1$) | **Agent 2** ($i = 2$) | **Agent 3** ($i = 3$) | **Agent 4** ($i = 4$) | **Agent 5** ($i = 5$) |
|---|---|---|---|---|---|---|
| $m_i$ | 2000 | 1200 | 1100 | 1500 | 1400 | 1500 |
| $g$ | 9.81 | 9.81 | 9.81 | 9.81 | 9.81 | 9.81 |
| $\alpha(s_i)$ | $0.05\sin(0.1s_0)$ | $0.05\sin(0.1s_1)$ | $0.05\sin(0.1s_2)$ | $0.05\sin(0.1s_3)$ | $0.05\sin(0.1s_4)$ | $0.05\sin(0.1s_5)$ |
| $w_i$ | Time-Varying | 5 | 5 | 5 | 5 | 5 |

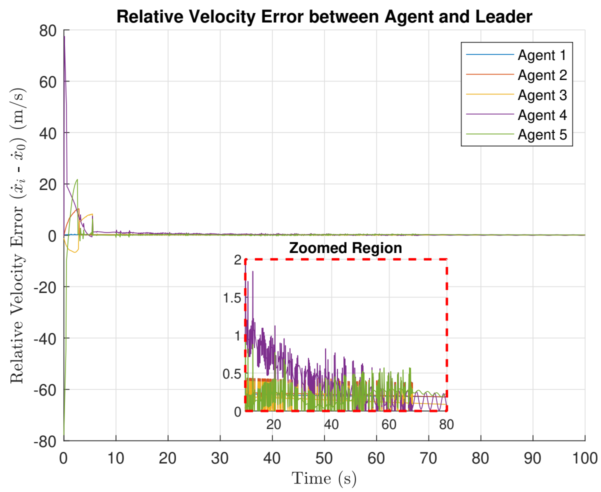


Fig. 5: Relative Velocity Error between Agents and Leader.

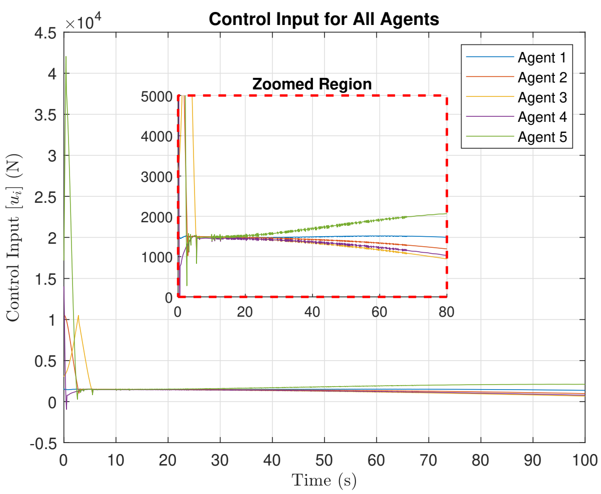


Fig. 6: Agents Control Inputs $u_i$, for $i \in \{1, 2, 3, 4, 5\}$.

## VI. CONCLUSION

This paper introduces a framework for high-order heterogeneous MAS with time-varying dynamics, integrating a unified control law, neural network-based parameter estimation, and adaptive consensus mechanisms. It ensures synchronization, stability, and collision/obstacle avoidance under fixed topologies using Lyapunov-based control laws and adaptive tuning, leveraging neural networks for real-time uncertainty handling and potential functions for safety. Numerical results validate the approach in a five-agent network with nonlinear dynamics. Future work aims to extend the framework to larger networks with dynamic topologies and communication delays.

## REFERENCES


[1] F. L. Lewis, H. Zhang, K. Hengster-Movric, and A. Das, *Cooperative Control of Multi-Agent Systems: Optimal and Adaptive Design Approaches*. Springer, 2013.

[2] W. Ren and R. Beard, “Consensus seeking in multiagent systems under dynamically changing interaction topologies,” *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, 2005.

[3] S. Khoo, L. Xie, and Z. Man, “Robust finite-time consensus tracking algorithm for multirobot systems,” *IEEE/ASME Transactions on Mechatronics*, vol. 14, pp. 219–228, 2009.

[4] Y. Hong, J. Hu, and L. Gao, “Tracking control for multi-agent consensus with an active leader and variable topologies,” *Automatica*, vol. 42, no. 7, pp. 1177–1182, 2006.

[5] J. Qin, H. Gao, and W. Wei, “Consensus control for multi-agent systems with switching topologies and time-varying delays,” *IEEE Transactions on Automatic Control*, vol. 56, no. 5, pp. 1245–1250, 2011.

[6] R. Olfati-Saber, “Flocking for multi-agent dynamic systems: Algorithms and theory,” *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[7] Z. Lin, B. Francis, and M. Maggiore, “Necessary and sufficient graphical conditions for formation control of unicycles,” *IEEE Transactions on Automatic Control*, vol. 50, no. 1, pp. 121–127, 2005.

[8] L. Moreau, “Stability of multi-agent systems with time-dependent communication links,” *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169–182, 2005.

[9] G. Shi and Y. Hong, “Global target aggregation and state agreement of nonlinear multi-agent systems with switching topologies,” *Automatica*, vol. 45, no. 5, pp. 1205–1211, 2009.

[10] J. Wang and M. Xin, “Multi-agent consensus algorithm with obstacle avoidance via optimal control approach,” in *Proceedings of the 2011 American Control Conference*, pp. 2783–2788, 2011.

[11] W. Ren, “Consensus based formation control strategies for multi-vehicle systems,” in *2006 American Control Conference*, pp. 6 pp.–, 2006.

[12] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, “Networked control systems: Analysis and design,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[13] A. Yesildirak, F. W. Lewis, and S. Jagannathan, *Neural Network Control of Robot Manipulators and Non-Linear Systems*. London, UK: Taylor & Francis, 1995.

[14] A. Koulong and A. Pakniyat, “Distributed adaptive consensus with obstacle and collision avoidance for networks of heterogeneous multi-agent systems,” *arXiv preprint arXiv:2410.08440*, 2024.